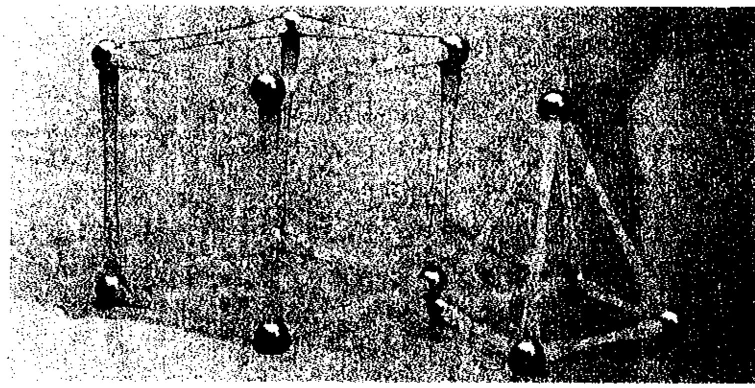
Problem-Solving Overview

Teachers must be prepared to approach mathematics instruction through questions, exploration, and problem solving.



According to research, two of the most important skills that students need in order to be prepared for 21st-century careers and citizenship are critical thinking and problem solving. Students need to be prepared to apply their knowledge and seek the right information in order to solve problems. "At the heart of critical thinking and problem solving is the ability to ask the right questions" (Wagner 2008). However, this ability is not inherent. Students must be taught how to question, and they must learn strategies that will help them solve problems, leading to more questions, more problems, and more solutions. And although the ability to vocalize their questions may not be innate, children are innately curious. This curiosity must be channeled and molded so that students can approach and solve problems in creative and meaningful ways.

According to the National Council of Teachers of Mathematics (NCTM 2000), students should be able to:

- build new mathematical knowledge through problem solving.
- solve problems that arise in mathematics and in other contexts.
- apply and adapt a variety of appropriate strategies to solve problems.
- monitor and reflect on the process of mathematical problem solving.

Teachers must be prepared to approach mathematics instruction through questions, exploration, and problem solving. In other words, problem solving is a way of teaching rather than the presentation of word problems. Exposing students to only traditional word problems is not sufficient. By doing so, they are given an unrealistic message about the way mathematics will serve them in the adult world. Most of the problems adults face require mathematical reasoning and skills that are not merely solved by translating the information given into mathematical sentences and then performing the necessary operations.

In order to function in a complex and changing society, it is necessary to be able to solve a wide variety of problems (Wagner 2008). In the real world, problems come in various shapes and forms, many of which involve mathematical concepts and applications. Often, there are numerous possible methods or strategies available to solve the problem. Students need to utilize all the resources they have developed, such as their knowledge, previous experience, and intuition. They then need to analyze, predict, make decisions, and evaluate the outcome of their solutions. For these reasons, it is extremely important that students have mathematics instruction that prepares them to become effective problem solvers.

Problem-Solving Overview (cont.)

Often, a greater emphasis is placed on algorithmic procedures, otherwise known as the arithmetic, because it has been more historically recognized and valued in society (Burns 2000). Arithmetic is ultimately necessary for solving many problems, but more emphasis needs to be placed on how it is used in real-life situations rather than on the actual computations. Problem solving is much more than finding answers to lists of exercises. It is, in essence, the ability to creatively approach, filter, and process information about a problem, and carry out a solution to that problem.

There are many classroom situations that lend themselves to illustrating real-life problems, such as collecting field-trip money, deciding on the number of buses needed for a field trip, taking attendance, and calculating grade averages. These are important and relevant ways to teach students problem-solving strategies. In addition, presenting students with contrived problem situations is also beneficial in building their problem-solving abilities because it furthers their understanding of specific problem-solving strategies. It is very challenging, but extremely important, that we offer motivating problems that spark children's natural curiosity, allowing them to use the skills they will need later (Burns 2000).

Problem-Solving Difficulty Factors

In addition to problem-solving strategies, it is important to have children recognize that the structure itself may pose a problem within the problem. There are seven difficulty factors that students must be able to recognize. Those difficulty factors are:

1. wrong order
2. key words
3. extra numbers
4. hidden word numbers
5. implied numbers
6. multiple steps
7. exact mathematical vocabulary

Each of these difficulties needs to be identified for the student, and students must have time to discover the difficulty factors within a problem. By recognizing this before attempting to solve the problem, the student is prepared to deal with the situation, and is less apt to be confused. These difficulty factors must be taught, and a list should be displayed in the classroom for all students to see.

Problem-Solving Overview *(cont.)*

Problem-Solving Difficulty Factors *(cont.)*

Listed below are the seven word problem difficulty factors and some recommendations when teaching them to your students.

1. Wrong Order

Numeral order is important when working subtraction and division problems. The earliest difficulty factor the learner encounters is the order in which the numerals appear in different problems. The following are examples:

Grades K–2 Example

Eric gave away 37 toy cars from his collection. He used to have 71 toy cars. How many toy cars does Eric have left?

Students often search for numbers and write them down in the order they appear in the problem. In this problem, the smaller number of 37 is given first. The student writes down this number. The second number given is 71, which the student writes under the 37. The question then calls for the operation of subtraction to be used, so the student proceeds to subtract 71 from 37. In most cases, the student does not recognize why he or she experiences difficulty solving the problem.

Grades 3–5 Example

Alexa's little brother weighs 36 pounds. Her dog, Fluffy, weighs 51.5 pounds. How many more pounds does her dog weigh than her little brother?

In this problem, the student not only is dealing with wrong order, but must also recognize that this problem involves comparison subtraction. The student assumes that the operation is addition because of the order of the numbers and the words *many* and *more*. Instead of recognizing the need to subtract, a student can easily be misled into thinking that this is a problem where something is increasing, and the comparison of the weights becomes a difficulty factor for the student.

Secondary Example

The football team drank $9\frac{3}{4}$ gallons of water at practice. The container holds 15 gallons of water. How much water is left in the container?

In this problem, the student is given the information in the opposite order in which it needs to be calculated. The student must use the larger number first in order to create the correct problem to solve.

Wrong Order Teaching Tip: Have students analyze the question first to determine what is asked. Teach students to write the equation once they understand what type of solution is needed.

Problem-Solving Overview *(cont.)*

Problem-Solving Difficulty Factors *(cont.)*

2. Key Words

Key words are taught in early elementary school but can be very misleading after primary grades. Students who depend on this strategy are easily fooled. The following is an example of three different situations that pose a problem to students who learn to solve word problems by merely searching for key words:

Grades K–2 Example

Maggie has 3 boxes of crayons. In every box, she has 12 crayons of all different colors. How many crayons does Maggie have altogether?

In this problem, a student who has been taught that the key words *have* and *altogether* mean *to add* is in trouble since the answer to this problem is not 15. Students need to recognize that there may be more than one operation associated with certain key words.

Grades 3–5 Example

Drew planned on getting off the top of a tower by climbing down a massive 1,000-foot rope. The tower rises 865 feet straight up. How much extra rope did Drew have?

In this situation, there is not a key word to help the child identify the operation needed to solve this problem.

Secondary Example

Shen paid Ricardo $20.75 for a combination of 13 baseball cards and some basketball cards. He paid $1.25 for each baseball card and $0.75 for each basketball card. How many cards did Shen buy altogether?

Key Words Teaching Tips: Show how key words can be misleading and stress the importance of reading the entire problem before solving it. The best suggestion is that key words are most helpful when they appear in a question immediately before the question mark. Also, point out that some key words might have more than one operation associated with them. An example of this is the word *altogether*. In the primary grades, this is taught with addition, and in the intermediate grades, it could be used for multiplication.

Problem-Solving Overview *(cont.)*

Problem-Solving Difficulty Factors *(cont.)*

3. Extra Numbers

Problems with too many numerals can be extremely difficult for students. They are uncertain which numerals should be used. The following are examples of this difficulty factor:

Grades K–2 Example

Cindy has 15 videos, Carlos has 7 videos, and Robert has 11 videos. How many more videos does Cindy have than Robert?

Grades 3–5 Example

Lee, Ana, and Hector collect marbles. Lee has 152 marbles in his collection. Ana has 149 marbles, and Hector has 126 marbles. How many more marbles does Lee have than Ana?

In both of these problems, there are three sets of numbers; however, one set is not needed to solve the problem.

Secondary Example

At a track competition, Amal competed in the long jump. He was given three attempts. On his first jump, he went 13 feet $8\frac{3}{4}$ inches. On his second jump, he went 13 feet $11\frac{1}{2}$ inches. On his last jump, he went 13 feet $8\frac{1}{4}$ inches. How much farther was his longest jump than his shortest jump?

Extra Numbers Teaching Tip: Teach students to deal with this difficulty factor by talking about numbers and their relationship to the story line. Discuss why the extra number(s) should be eliminated.

4. Hidden Word Numbers

Students often look for two numerals and key words in a problem. They give the problem very little thought. Writing one of these numbers in word form complicates this strategy. The following are examples:

Grades K–2 Example

Maseo had three baseball cards. Adam had 5 baseball cards, and Tanisha had 4. How many baseball cards did they have altogether?

Problem-Solving Overview *(cont.)*

Problem-Solving Difficulty Factors *(cont.)*

4. Hidden Word Numbers *(cont.)*

Grades 3–5 Example

A thousand thrill seekers came to watch the mountain climber rescue Drew. If the average thrill seeker spent $14.00 for the trip to the monument, how much money was spent for travel?

Secondary Example

Julianne has $20.00 to purchase a shirt that costs $15.50. When she was checking out, the salesperson told her that the shirt is discounted twenty percent. How much change will she get after purchasing the shirt?

Hidden Word Numbers Teaching Tip: Teach number words first. Once children can identify numbers written as words, give them time to practice locating numbers in context. When they find numbers, either written as words or numerals, have them highlight the words.

5. Implied Numbers

These problems are often related to problems that contain another difficulty factor: *Exact Mathematical Vocabulary* difficulty factor. The problem might not present enough information or one of the numbers necessary for solving the problem is implied in a term, such as a measurement term. One example is using the measurement word *foot* when children need to use 12 inches to solve the problem. The following are examples of problems with implied numbers:

Grades K–2 Example

Bradon ate a dozen crackers, Mina ate 6 crackers, and Salena ate 9 crackers. How many crackers were eaten in all?

The cracker problem is difficult if students do not know the numerical value of a dozen. Often, students see the word *a* and add 1 to the sum of 6 and 9.

Grades 3–5 Example

Amy needed milk for a dessert that she was going to make for the bake sale. She went to the store and bought 1 pint of milk. She used 1 cup of milk in the recipe. How much milk was left after she made the dessert?

In this problem, the student might compare 1 cup to 1 pint and assume all the milk was used. Here, the student must know how many cups are in a pint.

Problem-Solving Overview *(cont.)*

Problem-Solving Difficulty Factors *(cont.)*

5. Implied Numbers *(cont.)*

Secondary Example

Mrs. Chen's science class will be planting a butterfly garden that has an area of $157\frac{1}{2}$ square feet. The length of the area is 5 yards. What is the width of the area?

Implied Numbers Teaching Tips: Teach children to look for words containing implied numbers. Practice by highlighting them and converting them to match the context of the other numbers. Always have students check first to see if the implied numbers are needed to solve the problem and are not just extra information.

6. Multiple Steps

Problems with multiple steps are difficult for students. Often, they will complete only one step; or they complete both steps correctly, but their calculations were wrong in the first step, ultimately producing an incorrect solution.

Grades K–2 Example

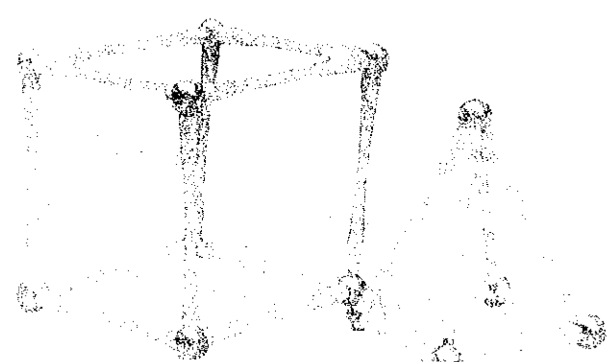
Alana has 8 gummy bears. She eats 3 of them. She then gives 2 to Bailey. How many gummy bears does she have left?

The best strategy is to act out these types of problems.

Grades 3–5 Example

Gretel picked 24 limes in the morning and 30 limes in the afternoon. Her grandmother gave her several bags and asked her to put 6 limes into each bag. How many bags will Gretel be able to fill?

Once a student identifies this as a multi-step problem, he or she must next determine the operations needed to solve it. In this problem, the student might first add all of the limes picked and divide them into groups of six or divide both numbers into groups of six and add the two quotients. Both strategies give the same result.



Problem-Solving Overview *(cont.)*

Problem-Solving Difficulty Factors *(cont.)*

6. Multiple Steps *(cont.)*

Secondary Example

Mr. Lin bought 3 picture frames for $64.50. If each picture frame cost $20.00 before tax was added, what tax rate did he pay for the 3 frames?

Multiple Steps Teaching Tip: Save problems of this type until after the students have mastered the other difficulty factors. Present the problems by demonstrating and displaying common steps needed to solve these types of problems. You may want to act out an example for clarification.

7. Exact Mathematical Vocabulary

Using exact terminology increases the difficulty of a problem. Students must be able to interpret the mathematical vocabulary to understand the situation. They must also be able to identify the possible equations associated with the terms to solve the problems. Two examples of exact mathematical vocabulary are *perimeter* and *mean*. The following are examples of problems with exact mathematical vocabulary:

Grades K–2 Example

What is the perimeter of a triangle where each side equals 3 inches?

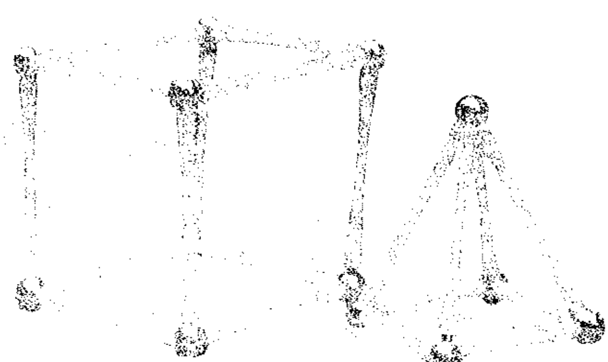
Grades 3–5 Example

Four classes raised money for a charity. Mr. Suarez's class raised $120.00, Ms. Barnes' class raised $62.00, Ms. Bard's class raised $80.00, and Mr. Feng's class raised $65.00. What is the mean amount of money raised?

Secondary Example

What is the surface area of a sphere that has a radius of 8 feet?

Exact Mathematical Vocabulary Teaching Tip: Identify common mathematical terms. Often, word problems use measurement units and geometric terms. Some examples are *circumference*, *perimeter*, and *area*.



Problem-Solving Overview *(cont.)*

Teaching About Difficulty Factors

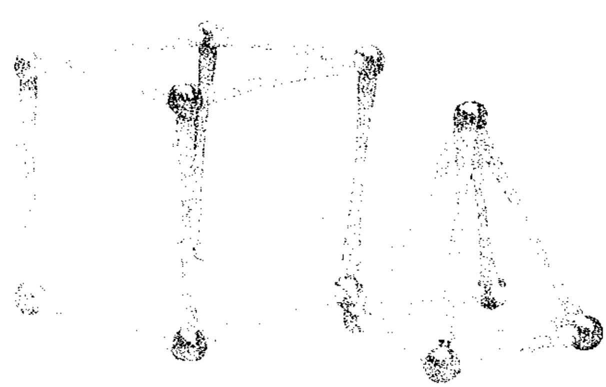
Each difficulty factor needs to be taught separately, and students need time to practice identifying which difficulty factor is present. The following procedure is a way to accomplish this:

- Introduce the difficulty factor.
- Give students several word problems, some with the difficulty factor just taught and some with no difficulty factors.
- There are two options for practice:
    1. Have students find all of the problems with the difficulty factor taught and highlight the part of the problem that has the difficulty factor. This helps them to focus their attention on the difficulty present and not necessarily on the operation to solve.
    2. Have students solve the problems with the difficulty factors only after they have identified them.
- Introduce a new difficulty factor only after students have mastered the current difficulty factor.
- As each new difficulty factor is introduced and students practice identifying them in the problem, students will then be able to handle problems that contain more than one difficulty factor. The student identifies which difficulty factor the problem has by highlighting it and writing the difficulty factor after the problem.
- For continued practice, as students enter the room each day, provide them with one problem containing a difficulty factor on a strip of paper. Have students highlight or identify the difficulty factor in the problem and then solve it. This practice is a great warm-up for your daily lesson.

Problem-Solving Process

When teaching problem solving, it is important to present students with a method of approaching all problems. For many students, the hardest part of problem solving is finding a starting point. George Polya (1973) proposed a four-stage approach to help children learn how to problem solve:

1. understanding the problem
2. devising a plan
3. carrying out the plan
4. looking back



Problem-Solving Overview *(cont.)*

Problem-Solving Process *(cont.)*

Polya's method is a systematic approach to problem solving that provides a guideline for how a problem solver moves through the process of solving many types of problems.

1. Understanding the Problem

What is it?

Understanding the problem involves interpreting what the problem means, and what questions must be answered to solve it. Students need to understand a problem thoroughly so they can determine what question is being asked in order to solve the problem and restate it in their own words.

What does it look like in the classroom?

- Students should read the problem carefully until they understand what is happening in the problem and what information is needed to solve the problem.
- Students should underline or highlight unfamiliar words, then identify the meanings of those words.
- Students should locate and discard unneeded information. They also need to uncover missing information in order to solve the problem.
- Students should ask themselves questions such as, "What is the problem asking me to do?" and "What information is important for solving the problem?"
- Allow the students to have discussions with others, restating the problem in their own words if necessary.
- If students have difficulty reading or understanding the problem, they may need to dissect the problem sentence by sentence, focusing on one sentence at a time.

2. Devising a Plan

What is it?

To devise a plan means the student must choose a strategy that will help him or her solve the problem. In many problems, there is more than one applicable strategy. The student must choose the one strategy that makes sense to him or her and will help the most with solving the problem.

Problem-Solving Overview (cont.)

Problem-Solving Process (cont.)

2. Devising a Plan (cont.)

What does it look like in the classroom?

- Students should make a list of possible ways to solve the problem.
- Students should analyze their lists and choose one strategy they think will help them solve the problem most efficiently.
- Students should discuss their lists with others if they have difficulty choosing the best strategy.

3. Carrying Out the Plan

What is it?

To carry out the plan means to solve the problem. Strong instruction on various problem-solving strategies should be provided so that students can successfully follow through with a solution. Students must work through each part of the problem, using the strategy they chose, until they find the answer to the problem.

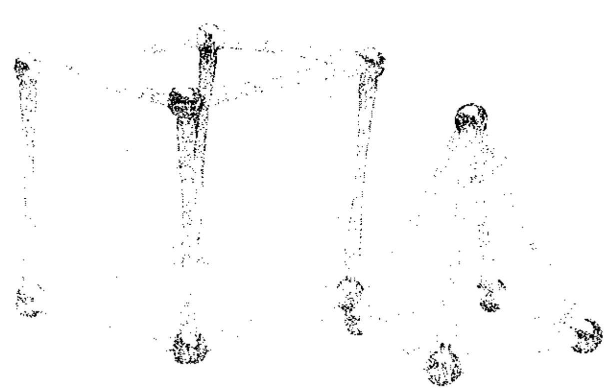
What does it look like in the classroom?

- Students should record their problem-solving steps in an organized manner so that they can see their work and decide if the strategy they selected will give them the needed results.
- If students get stuck, they can review their work to make sure they have not made any errors in their calculations.
- If the chosen strategy does not work with the problem, students should go back to their lists of strategies and choose a different approach to solve the problem.

4. Looking Back

What is it?

To look back is to examine the solution obtained from solving the problem using the chosen strategy. This step in the problem-solving process encourages students to reflect on the strategies they chose and make generalizations that could be applicable to future problems.



Problem-Solving Overview *(cont.)*

Problem-Solving Process *(cont.)*

4. Looking back *(cont.)*

What does it look like in the classroom?

- Students must reread the problem and check the solution to see if it meets the conditions stated in the problem and answers the question adequately.
- Students must ask themselves questions such as, "Does my solution make sense?" and "Is my solution logical and reasonable?"
- Students should illustrate or write down their thinking processes, estimations, and approaches. This will help them visualize the steps they took to solve the problem and make generalizations about their work.
- Allow students the opportunity to discuss with others and orally demonstrate or explain how they reached their solutions.
- Students should consider whether it is possible for them to have solved the problem in a simpler way.

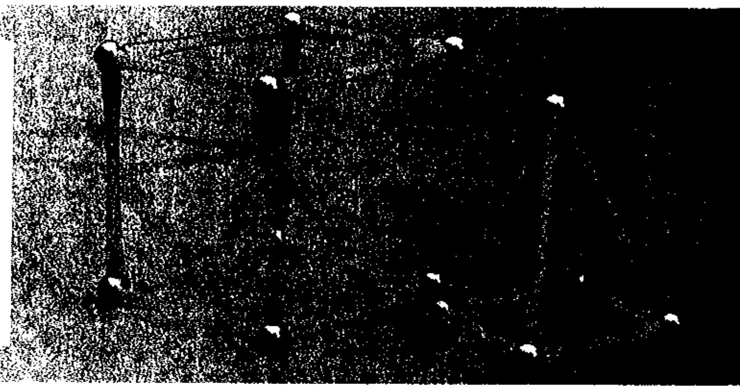
12 Strategies for Problem Solving

It is also important for teachers to provide students with specific strategies with which they can apply Polya's problem-solving approach. The following 12 strategies can be applied to a large variety of problem-solving situations:

- Drawing a Diagram
- Acting It Out or Using Concrete Materials
- Creating a Table
- Looking for a Pattern
- Guessing and Checking
- Creating an Organized List
- Working Backwards
- Creating a Tree Diagram
- Using Simpler Numbers
- Using Logical Reasoning
- Analyzing and Investigating
- Solving Open-Ended Problems

The sections that follow will provide background information on each strategy, procedures for teaching each strategy, and model problems and solutions at each grade-level range.

Drawing a Diagram



Standard

- uses a variety of strategies in the problem-solving process

Background Information

Drawing a diagram is a visual way of processing the information in a problem. It allows students to see what is happening and relate to the situation more clearly.

This strategy often reveals aspects of a problem that may not be apparent at first. If the steps or situations being described in a problem are difficult to visualize, using a diagram may enable the students to see the information more clearly.

There are many types of diagrams that can be used with this strategy. Some of those diagrams include drawing a picture, using symbols or lines to represent objects, and using a time/distance line.

Procedure

Once it is decided that drawing a diagram is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Decide what type of diagram will best show the information in the problem.
3. Draw the diagram according to the scenario in the problem.
4. Check your diagram and solution.
5. Record the solution.

Samples

The following skills and concepts illustrate how drawing a diagram can be used with many different types of problems. Students should be comfortable with them in order to be able to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

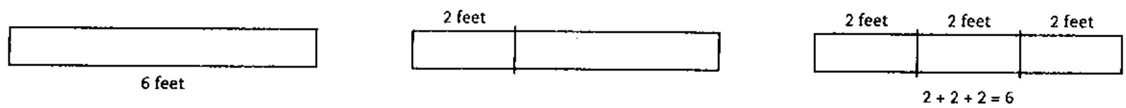
Drawing a Diagram *(cont.)*

Drawing a Picture

Drawing a picture can help students visualize the problem clearly and organize their thoughts.

Grades K–2 Sample

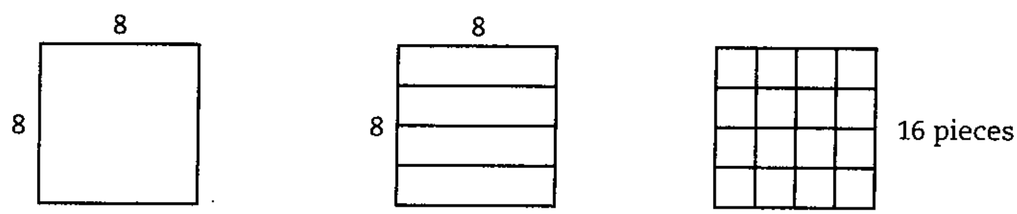
Eva has a ribbon. It is 6 feet long. She wants to cut the ribbon into pieces. She wants each piece to be 2 feet long. How many pieces of ribbon can she cut?



(step 1) (step 2) (step 3)

Grades 3–5 Sample

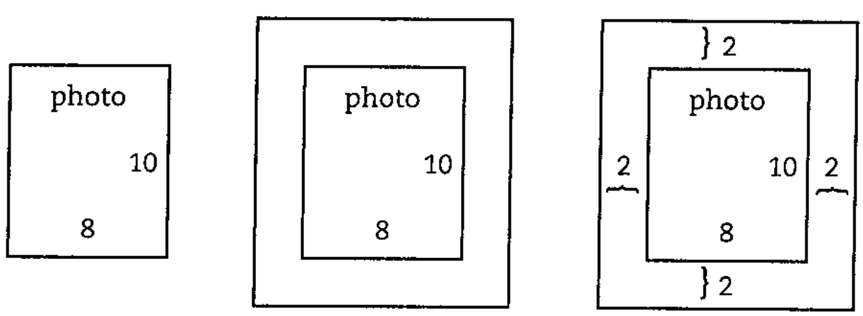
Miguel baked a cake for his mom's birthday. The rectangular pan he used measured 8" x 8". If he cuts 2-inch pieces, how many pieces can he cut?



(step 1) (step 2) (step 3)

Secondary Sample

Shen wants to place a border around a picture of his dad and himself. The picture is 8" wide and 10" tall. The border needs to be 2 inches wider and taller than the picture. What are the dimensions of the border?



width
2 + 8 + 2 = 12 inches

height
2 + 10 + 2 = 14 inches

(step 1) (step 2) (step 3) (step 4)

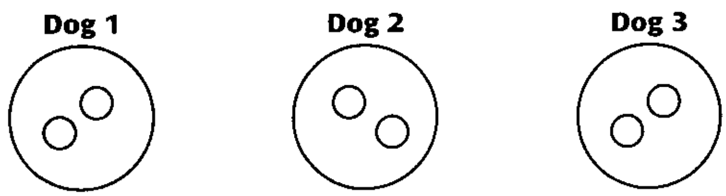
Drawing a Diagram *(cont.)*

Using Symbols or Lines to Represent Objects

Often, a problem discusses some type of object that could be difficult and time consuming to draw. Students can use lines or symbols, such as circles, squares, or triangles to represent the objects in the problem.

Grades K–2 Sample

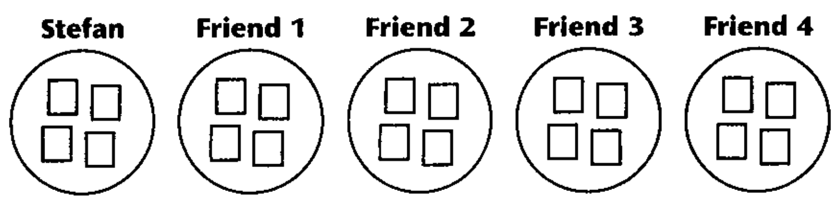
Kiara has 3 dogs. She has 6 balls that she wants to share equally among the dogs while they play. How many balls will each dog get to play with?



Each dog gets 2 balls.

Grades 3–5 Sample

Stefan has 23 trading cards that he wants to divide equally among himself and 4 friends. How many trading cards will each friend get? Are there any cards left over?

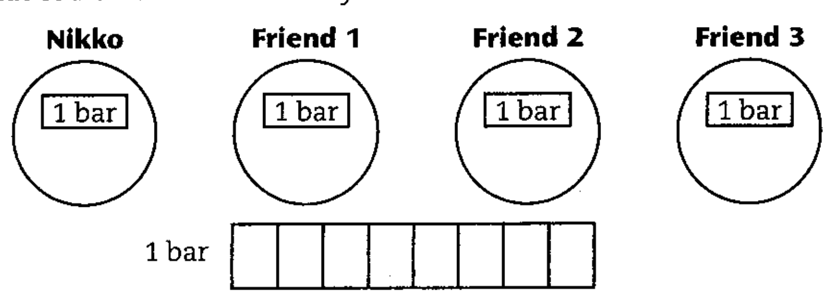


Each friend gets 4 cards. There are 3 cards leftover.

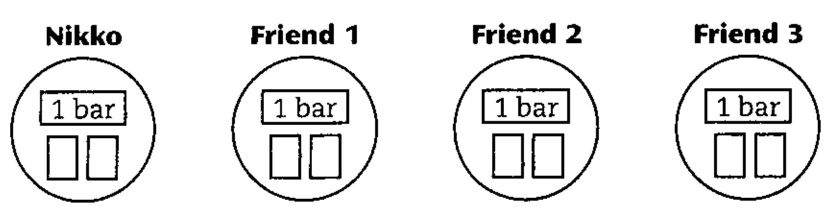
Secondary Sample

Nikko has 5 chocolate bars. Each chocolate bar can be divided into 8 parts. Nikko wants to share the chocolate evenly among himself and his 3 friends. How much chocolate does each person receive? Write a mathematical equation that could be used to check your work.

(step 1)



(step 2)



(step 3) Each person receives 1 bar and 2 pieces of chocolate. $4x = 5$

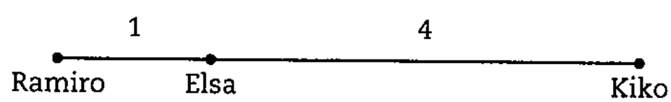
Drawing a Diagram *(cont.)*

Using a Time/Distance Line

A time/distance line helps to show distance or movement from one point to another.

Grades K–2 Sample

Ramiro, Elsa, and Kiko live on the same street. Ramiro's house is first on the street. Elsa lives 1 block down the street from Ramiro. Kiko lives 4 blocks down the street from Elsa. How many blocks is it from Ramiro's house to Kiko's house?

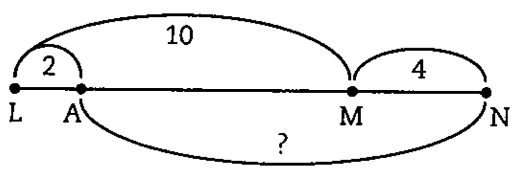


1 + 4 = 5 blocks

(step 1) (step 2)

Grades 3–5 Sample

Lina, Marco, Nori, and Aleyah live on the same street. Lina's house is 10 blocks west of Marco's house. Nori's house is 4 blocks east of Marco's house. Aleyah's house is 2 blocks east of Lina's house. How many blocks is it from Aleyah's house to Nori's house?



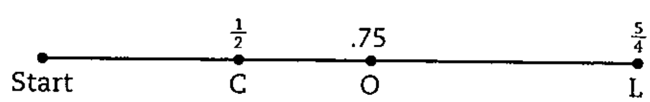
10 + 4 – 2 = 12 blocks

(step 1) (step 2)

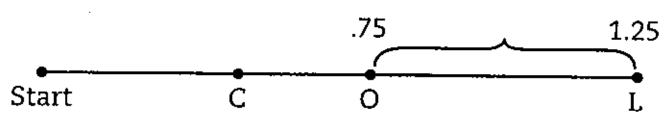
Secondary Sample

Carlos, Lilah, and Omar all had to run during P.E. Carlos ran $\frac{1}{2}$ of a mile. Omar ran .75 of a mile. Lilah ran $\frac{5}{4}$ miles. How much farther did Lilah run than Omar?

(step 1)



(step 2)



1.25 – .75 = .5 miles more

Acting It Out or Using Concrete Materials

Standard

- uses a variety of strategies in the problem-solving process

Background Information

This strategy is useful in helping students to gain a concrete understanding of an abstract problem. A variety of concrete objects such as counters, blocks, or beans can be used to represent the people or things in a problem. These objects can be moved or rearranged to fit the scenario or follow the steps in the problem.

There are many types of problems that can be solved using this strategy. Some of these types of problems include using amounts of money, specific quantities, position changes, and large numbers.

Procedure

Once it is decided that acting it out or using concrete materials is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Decide whether the problem is best solved by acting it out or by using concrete materials.
3. Gather the materials or actors.
4. Follow the steps of the problem using the materials or actors.
5. Record the solution based on the outcome of the scenario.

Samples

The following skills and concepts illustrate how acting it out or using concrete materials can be applied to many different types of problems. Students should understand and practice how to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Acting It Out or Using Concrete Materials (cont.)

Amounts of Money Problems

Often, problems require the use of money, giving change, or finding combinations of coins and bills that can be used to purchase items. This can be very confusing without the use of pretend money and acting out the scenarios.

Using the concrete material, pretend money, and acting out each step of the problem will help students arrive at the correct solution.

Grades K–2 Sample

Yelena had 8 quarters in her pocket. She lost 2 of the quarters while playing at recess. How much money does she have left?

Grades 3–5 Sample

Salena had a $20 bill. She exchanged the bill for two $10 bills. She took one of the $10 bills and exchanged it for two $5 bills. She took one of the $5 bills and exchanged it for five $1 bills. She took two of the $1 bills and exchanged them for 8 quarters. How many bills and coins does she have now?

Secondary Sample

Gabby works at a local toy store. The manager does not allow his employees to use calculators to give back change. A customer purchased toys totaling $13.47. The customer gave Gabby a $20 bill. How much change did the customer receive?

Specific Quantities

Sometimes quantities are given in a problem. Most often these problems can be acted out using the students in the class. However, using concrete materials may be more appropriate if the quantities are large.

Grades K–2 Sample

There were 4 birds sitting in a tree. Two of the birds flew away. Then 5 more birds came to sit in the tree. How many birds are sitting in the tree?

Grades 3–5 Sample

A city bus is traveling downtown. There are 36 people on the bus. At the first stop, one-third of the people get off of the bus and six people get on the bus. At the second stop, half of the people get off of the bus. At the third stop, eight people get off of the bus and two people get on the bus. How many people are on the bus now?

This problem has a high quantity of people and it may not be feasible to act this out using the students in class. Students can solve this problem using counters (or any other concrete material) as a model for the people on the bus.

Acting It Out or Using Concrete Materials (cont.)

Specific Quantities (cont.)

Secondary Sample

Many types of pie were served at a large holiday party. There were 10 pies at the beginning of the party. Right away, $2\frac{1}{2}$ pies were eaten because people were hungry. After dinner, half of the remaining pie was eaten for dessert. In the evening, an aunt came late to the party and brought 2 more pies. After the party, the hosts ate $\frac{1}{4}$ of a pie as a midnight snack. How much pie was left the day after the party?

Moving Positions

Problems can be confusing when the characters or objects are frequently moving around. By getting students to act out the problem as a group or by using concrete materials, the movements can be clearly visualized and organized.

Grades K–2 Sample

Lilah is walking around her backyard. She walks 10 steps forward. Then she walks 8 steps to the right. Next, she walks 6 steps backwards. Last, she walks 4 steps to the left. How many steps has she taken?

Grades 3–5 Sample

Moira is late meeting her friends at the fair. There are 40 people in line ahead of her for a ticket and she is very impatient. Each time a person in front of her is served, she cuts ahead of 3 people. How many people will be served before Moira reaches the front of the line?

Because this problem has a high quantity of people, students can use counters (or any other concrete material) as a model for the people waiting in line.

Secondary Sample

A football team started on its own 43-yard line with only a minute left in the game. They ran 5 plays before kicking a field goal to win the game. During the first possession, they gained 10 yards. The second possession, the team had a loss of 4 yards. The third possession, they gained 7 yards. The fourth possession, they had a loss of 3 yards. The fifth possession, they gained 11 yards. From which yard line did they kick the field goal?

Acting It Out or Using Concrete Materials *(cont.)*

Using Large Numbers

When a problem contains large numbers, it is often most practical for students to use concrete materials to solve it. Students can use any concrete materials, such as blocks, counters, beans, or small manipulatives, to represent the objects in the problem.

Grades K–2 Sample

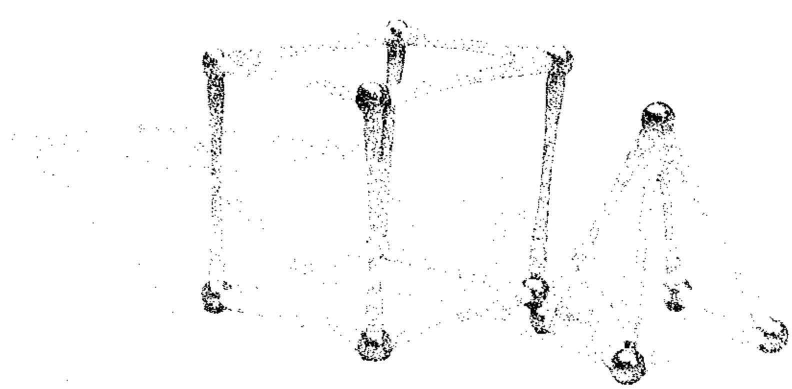
On the 100th day of school, Adrian brought in 100 cookies to share with his classmates. If there are 20 students in the class, how many cookies does each student get?

Grades 3–5 Sample

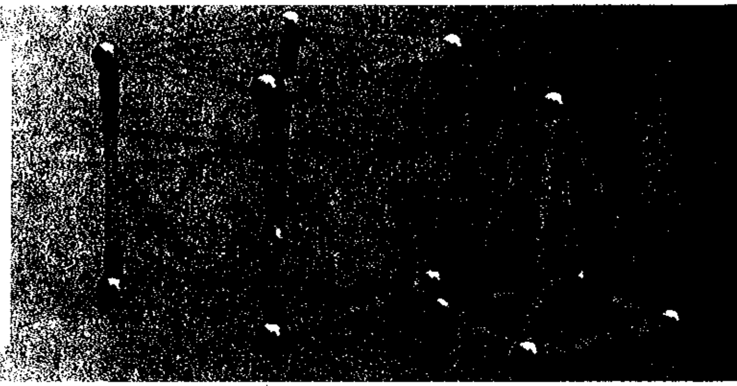
There were 60 chocolates in a row waiting to be boxed in the chocolate factory. Shen's job is to taste the chocolates to make sure they taste right. Starting with the 8th chocolate, he tastes every 9th one. How many chocolates does Shen taste?

Secondary Sample

Mrs. DeLeon has 14 students that play in a tennis tournament two times a week after school. Each student will be paired with one of the other students to play a game. The games will continue until every student has played each of the other students one time. How many games will be played in the tournament?



Creating a Table



Standard

- uses a variety of strategies in the problem-solving process

Background Information

A table often helps to organize information from a problem so that it can be easily understood and relationships can be clearly seen. This is a good strategy to use when a problem contains information with more than one characteristic. By creating a table, it is easy to see what information you have and what information is missing. Using a table can also reduce the possibility of making mistakes or repetitions.

There are many types of problems that can be solved using this strategy. Some of these types of problems include calculating multiples and following patterns.

Procedure

Once it is decided that creating a table is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Decide what information is known in the problem.
3. Use the information from the problem to choose the number of columns needed for the table and to decide the headings for each column.
4. Use the information from the problem to choose the number of rows needed for the table.
5. Create the table.
6. Fill in the known information in the table.
7. Continue filling in information in the table until the solution to the problem is reached.
8. Check the work and record the solution.

Samples

The following skills and concepts illustrate how creating a table can be used with many different types of problems. Students should be comfortable with these skills to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Creating a Table *(cont.)*

Calculating Multiples

Sometimes problems use multiples of numbers. When this information is put into a table, the pattern can be quickly seen. Then, the information can be easily used to solve the problem.

Grades K–2 Sample

Mr. Lin's class is making holiday cards to give to people at the local retirement home. Each day, the class makes 5 cards. How many cards will they have after 10 days?

Day	1	2	3	4	5	6	7	8	9	10
Cards	5	10	15	20	25	30	35	40	45	50

Grades 3–5 Sample

Maria and Celine planted a flower garden at the school. On Monday, they each planted two flowers. Every day, Maria and Celine each planted twice as many flowers as they did the day before. How many flowers did the girls plant during the week?

	Monday	[illegible]	Wed.	Thursday	[illegible]	[illegible]
[illegible]	2	4	8	16	32	62
[illegible]	2	4	8	16	32	62
[illegible]	4	8	16	32	64	124

Secondary Sample

Brody's dad told him that he could choose how he could get his allowance. With the first option, he could receive $17.00 each month. With the second option, he would get $0.57 in January, and the amount would double each month after that. Which option do you think is the better deal? Create a table showing how much he would receive each month with both options. Calculate the total with each option.

Option 1 $17.00 × 12 = $204.00

Option 2 (see chart below)

Month	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.
Amount	$0.57	$1.14	$2.28	$4.56	$9.12	$18.24	$36.48	$72.96	$145.92	$291.84	$583.60	$1,167.36

Total: $2,334.07

Creating a Table (cont.)

Following Patterns

Once a table is created, numbers are filled in using the information from the problem. Sometimes a pattern can be found once the numbers are inserted in a table. The pattern can be used to solve the problem.

Grades K–2 Sample

Ms. Medina's class went on a field trip to the farm. They counted the chickens in 4 different pens. In the first pen, there were 3 chickens. In the second pen, there were 4 chickens. In the third pen, there were 5 chickens. In the fourth pen, there were 6 chickens. Each chicken has 2 feet. How many total chicken feet were there in each of the pens?

Number of Chickens	3	4	5	6
Number of Feet	6	8	10	12

Grades 3–5 Sample

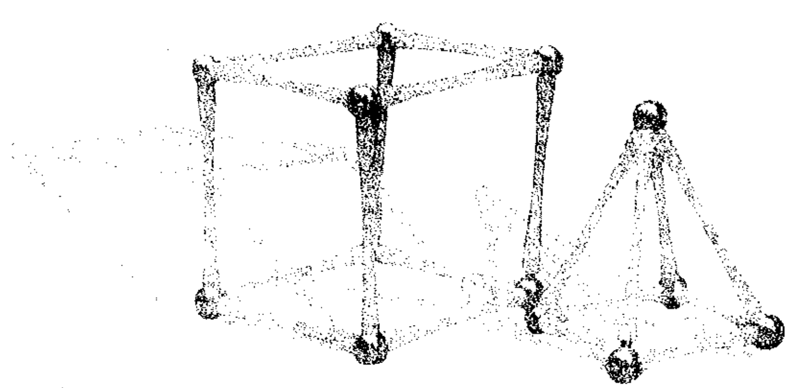
One morning the temperature was 6° celsius at 8:00 A.M. The students recorded the temperature every hour. At 9:00 A.M., the temperature was 8° celsius. At 10:00 A.M., the temperature was 12° celsius. At 11:00 A.M., the temperature was 18° celsius. If this pattern continues, what will the temperature be at 2:00 P.M.?

Time	8:00	9:00	10:00	11:00	12:00	1:00	2:00
Temperature	6°	8°	12°	18°	26°	36°	48°

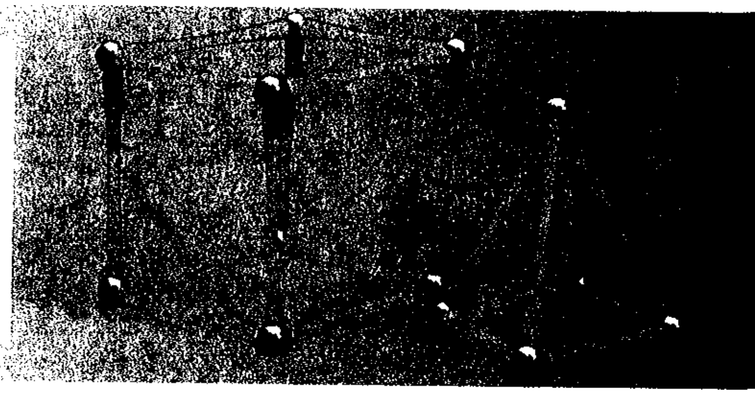
Secondary Sample

Andrew and Michael are trying to break their trampoline-jumping record from last year. The boys take turns jumping for an hour. Michael consistently makes 15% fewer jumps than Andrew does. Andrew increases the number of jumps he makes in an hour by 15 each time he jumps. During Andrew's first hour, he jumps 150 times. Based on this, how many jumps did Andrew make during his fifth hour on the trampoline?

Hour	1	2	3	4	5
Jumps	150	165	180	195	210



Looking for a Pattern



Standard

- uses a variety of strategies in the problem-solving process

Background Information

Since mathematical patterns can be found in numbers, shapes, nature, and the world around us, looking for a pattern is one of the most frequently used problem-solving strategies. This strategy is an extension of the Creating a Table strategy and the Creating an Organized List strategy. Once information is organized, it is easier to see if a pattern exists and find the "rule."

There are several ways to check for a pattern in a problem and determine its rule:

- Determine if the numbers are increasing or decreasing by a regular sequence.
- Determine the difference between two consecutive numbers.
- Determine whether the numbers have been divided or multiplied by any given number.

Once students have determined the pattern in a problem, they should be able to continue the pattern to find a given unit within it.

There are many types of problems that can be solved using this strategy. Some of those types of problems include using spatial patterns and using tables.

Procedure

Once it is decided that looking for a pattern is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Organize the information in the problem into a table or a list. This will help students see how the information increases or decreases.
3. Look for a repeated sequence in the pattern to determine the rule for the pattern. If a repeated sequence is found, then the rule has been found.
4. If no repeated sequence exists, find the difference between the first two numbers in the pattern. Calculate the difference between other numbers in the pattern to see if it is the same. If it is the same, than the "rule" has been found.

Looking for a Pattern *(cont.)*

Procedure *(cont.)*

5. If no uniform pattern exists, look for a pattern using multiplication or division.
6. If no multiplication or division pattern exists, look for an increasing or decreasing pattern (e.g., +1, +2, +3, etc.).
7. Once the pattern is determined, reread the problem to see what information is needed from the pattern.
8. Check the work and record the solution.

Samples

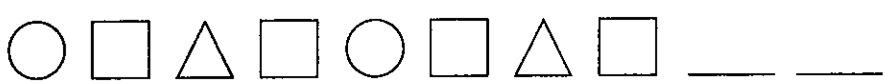
The following skills and concepts illustrate how looking for a pattern can be used with many different types of problems. Students should be comfortable with them in order to be able to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Spatial Patterns

Spatial patterns involve shapes and patterns, textures, or designs within shapes. These types of patterns are most used in elementary school, but can be appropriate for secondary-level students as functions are introduced. Spatial patterns can be repetitive or growing, depending on the ability levels of the students.

Grades K–2 Sample

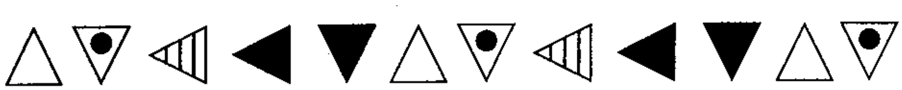
What are the next 2 shapes in this pattern? What is the pattern unit?



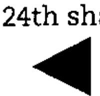
next shapes:  **pattern unit:** 

Grades 3–5 Sample

Continue this pattern to find the 24th shape. Create a pattern of your own that is similar to this one.



24th shape:

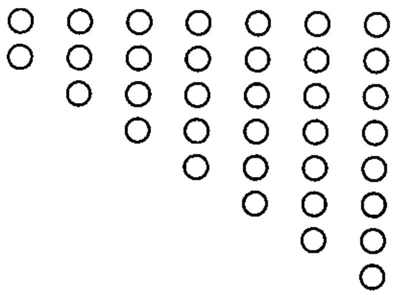


Looking for a Pattern *(cont.)*

Spatial Patterns *(cont.)*

Secondary Sample

Look at the pattern below. How many circles would be in the 50th unit in this pattern? What equation did you use to solve this problem?



50th unit has 51 circles

$x + 1$

Finding a Pattern in a Table

Often, students are given a table of data or information that can be used to fill in the missing pieces of that table. Students must analyze the information in the rows and columns and try different strategies (finding the difference, finding the growth using multiplication) for finding the pattern within the numbers. Once the pattern is found, the missing information can be calculated and recorded in the proper places within the table.

Grades K–2 Sample

Rory has a garden in his backyard. He picks tomatoes every day. On Monday he picked 2 tomatoes. On Tuesday, he picked 4 tomatoes. On Wednesday, he picked 6 tomatoes. If this pattern continues, how many tomatoes will he pick on Friday?

Day of the Week	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Tomatoes Picked	2	4	6	8	**10**

Grades 3–5 Sample

Each school day Ernesto saves 2 cookies from his lunch. After 10 school days, how many total cookies will he have saved?

[illegible]	1	2	3	4	5	6	7	8	9	10
Cookies Saved	2	2	2	2	2	2	2	2	2	2
Total Cookies Saved	2	4	6	8	10	12	14	16	18	**20**

Looking for a Pattern *(cont.)*

Finding a Pattern in a Table *(cont.)*

Secondary Sample

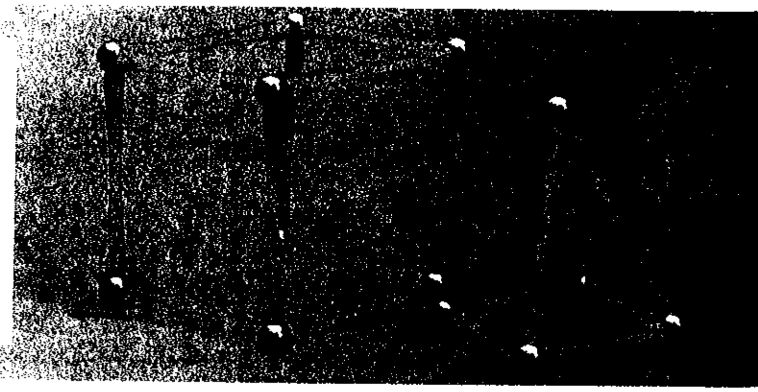
For Cho's birthday, her grandfather wants to give her money to buy some new clothes. He has agreed to give her money each day during the month of her birthday, but the amount depends on a special rule. He has decided to double the amount of money he gives her each day for 15 days, starting with $0.01 on the first day. He will only give Cho the money if she can accurately tell him how much total money she should receive from him by following this rule. How much money will Cho receive in total after the 15 days are up?

Days	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Amount Received	$0.01	$0.02	$0.04	$0.08	$0.16	$0.32	$0.64	$1.28	$2.56	$5.12	$10.24	$20.48	$40.96	$81.92	$163.84
Total	$0.01	$0.03	$0.07	$0.15	$0.31	$0.63	$1.27	$2.55	$5.11	$10.23	$20.47	$40.95	$81.91	$163.83	**$327.67**

Guessing and Checking

Standard

- uses a variety of strategies in the problem-solving process



Background Information

Guessing and checking is a strategy that is often overlooked by teachers. So often students are told that guessing is not a good strategy. What students need to understand is that *wild* and *random* guessing is not a good strategy. Making an *educated guess* about a solution to a problem is a valuable strategy and is a good tool to use even outside of mathematics.

In this strategy, students make an educated guess as to the solution of the problem based on the information they are given. Then they check their guess against the conditions of the problem, evaluate the results, and make another guess according to the results of the previous guess. This process is repeated until the correct solution is found.

There are many types of problems that can be solved using this strategy. Some of these types of problems include using objects and using time or distance.

Procedure

Once it is decided that guessing and checking is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Note the important facts in the problem and determine the exact problem that needs to be solved.
3. Help students create a table in which to record their guesses. Tell them that a table is a good way to keep their guesses organized.
4. All students should then make an initial guess. Remind them that their guesses should be reasonable and based on the important information from the problem.
5. Record the solution to the first guess. Have students evaluate the answer. They should ask themselves if their answer correctly solves the problem.
6. If their answers do not correctly solve the problem, have them look at their original guess again. Based on their answer, they should decide whether their second guess should be higher or lower than their first guess.

Guessing and Checking *(cont.)*

Procedure *(cont.)*

7. Students should repeat steps 4–6 until the correct answer is found.
8. Check the work and record the final answer.

Samples

The following skills and concepts illustrate how guessing and checking can be used with many different types of problems. Students should be comfortable with them in order to be able to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Using Objects

Grades K–2 Sample

Carlos and Ami have each lost some teeth. Together, they have lost 10 teeth. Ami has lost 2 more teeth than Carlos. How many teeth has Carlos lost? How many teeth has Ami lost?

Guess	1	2	3	4	5	6
Carlos						
Ami						
Total Teeth						

Grades 3–5 Sample

Kiara, Dimitri, and Ravi have each made a photo book of their friends. Together they have 260 photos. Dimitri has 25 more photos than Ravi. Kiara has 30 more photos than Dimitri. How many photos are in each person's photo book?

Guess	1	2	3	4	5	6
Kiara						
Dimitri						
Ravi						
Total Photos						

Secondary Sample

Ling has $6.00 in dimes, nickels, and quarters in her bank. She has double the number of dimes than nickels. The number of quarters is one-fourth the number of dimes. How many of each does she have in her bank?

Guess	1	2	3	4	5	6
Dimes						
Nickels						
Quarters						
Total						

Guessing and Checking *(cont.)*

Using Time/Distance

Grades K–2 Sample

Nina, Ana, and Tess are sisters. Nina is the youngest. Ana is 2 years older than Nina. Tess is 5 years older than Ana. Their age totals 30. What are their ages?

Guess	1	2	3	4	5	[illegible]
[illegible]						
[illegible]						
[illegible]						
[illegible]						

Grades 3–5 Sample

The Granger family is driving to their grandparents' house for the holidays. Their grandparents' house is 1,500 kilometers away, and they decide to split the drive into 4 days so that they can sightsee along the way. Each day they travel 20 kilometers more than the previous day. How many kilometers do they travel each day?

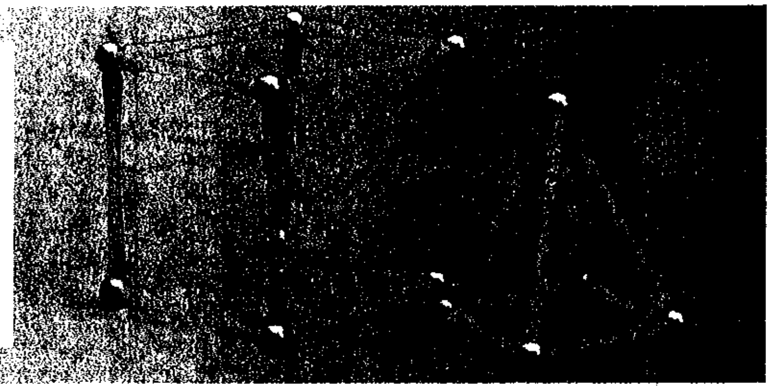
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]						
[illegible]						
[illegible]						
[illegible]						
[illegible]						

Secondary Sample

Lilah, Nihal, and Kiko ran a mile in P.E. They finished with a combined time of 32.5 minutes. Lilah finished in half the time that Kiko finished. Nihal finished in two-thirds the time that Kiko finished. How long did it take each of them to run a mile?

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Lilah						
Nihal						
Kiko						
Total Time						

Creating an Organized List



Standard

- uses a variety of strategies in the problem-solving process

Background Information

Creating an organized list is a similar strategy to creating a table. Often, this strategy is used instead of a table when there is more information in the problem. This strategy is different than using a table because it allows the problem solver to arrange the information more systematically so that the answer is clear. Students should follow a sequence or procedure to make sure that all possibilities are tested or found. Creating a list ensures that no information is duplicated.

There are many types of problems that can be solved using this strategy. Some of those types of problems include sequencing and number combinations.

Procedure

Once it is decided that creating an organized list is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Decide what information is known in the problem.
3. Decide what information should be kept the same as you work through the problem.
4. Help students create combinations and work sequentially to list the possible solutions to the problem.
5. Reread the problem and look back at the list to make sure no information is missing or repeated.
6. Record the solution.

Samples

The following skills and concepts illustrate how creating an organized list can be used with many different types of problems. Students should be comfortable with these skills to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Creating an Organized List *(cont.)*

Sequencing

Often, lists need to be completed in a specific order so that all of the combinations or pieces are written down. Sequencing items in the correct order guarantees that no information is omitted.

Grades K–2 Sample

Marisol has three picture frames. One frame is yellow, one frame is blue, and one frame is green. She wants to put them on top of the bookcase in her room. How many different ways could she put them on top of the bookcase?

Y	B	G	B	G	Y	G	B	Y	6 different ways
Y	G	B	B	Y	G	G	Y	B	

Grades 3–5 Sample

Tia has two choices for ice cream: chocolate and vanilla. She has three toppings: peanuts, fudge, and sprinkles. How many combinations of ice cream and toppings can she choose from?

chocolate—peanuts	vanilla—peanuts	
chocolate—fudge	vanilla—fudge	6 different combinations
chocolate—sprinkles	vanilla—sprinkles	

Secondary Sample

The basketball team is getting new uniforms. They can choose from three colors for the jerseys: red, blue, or silver. They can choose from two colors for the shorts: black or white. They can choose from two colors, black or white, for the numbers on the jerseys. How many combinations can be made for the new uniforms?

Jerseys	Shorts	Numbers	
red	black	black	
red	black	white	
red	white	black	
red	white	white	
blue	black	black	
blue	black	white	12 uniform combinations
blue	white	black	
blue	white	white	
silver	black	black	
silver	black	white	
silver	white	black	
silver	white	white	

Creating an Organized List *(cont.)*

Number Combinations

Sometimes problems call for students to put numbers together in specific combinations. It is also important to make sure that this is done systematically so that nothing is repeated or omitted.

Grades K–2 Sample

How many three-digit numbers can be made using the numerals 4, 8, and 2?

482	824	284	6 different numbers
428	842	248	

Grades 3–5 Sample

How many addition problems can you create using the digits 7, 2, 3, and 6, with a sum between 0 and 100?

$$\begin{array}{r} 23 \\ +\,76 \\ \hline 99 \end{array}\quad \begin{array}{r} 23 \\ +\,67 \\ \hline 90 \end{array}\quad \cancel{\begin{array}{r} 32 \\ +\,76 \\ \hline 108 \end{array}}\quad \begin{array}{r} 32 \\ +\,67 \\ \hline 99 \end{array}\quad \begin{array}{r} 27 \\ +\,36 \\ \hline 63 \end{array}\quad \begin{array}{r} 27 \\ +\,63 \\ \hline 90 \end{array}\quad \cancel{\begin{array}{r} 72 \\ +\,36 \\ \hline 108 \end{array}}\quad \cancel{\begin{array}{r} 72 \\ +\,63 \\ \hline 135 \end{array}}\quad \begin{array}{r} 26 \\ +\,73 \\ \hline 99 \end{array}\quad \begin{array}{r} 26 \\ +\,37 \\ \hline 63 \end{array}\quad \cancel{\begin{array}{r} 62 \\ +\,73 \\ \hline 135 \end{array}}$$

$$\begin{array}{r} 62 \\ +\,37 \\ \hline 99 \end{array}\quad \begin{array}{r} 37 \\ +\,26 \\ \hline 63 \end{array}\quad \begin{array}{r} 37 \\ +\,62 \\ \hline 99 \end{array}\quad \begin{array}{r} 73 \\ +\,26 \\ \hline 99 \end{array}\quad \cancel{\begin{array}{r} 73 \\ +\,62 \\ \hline 135 \end{array}}\quad \cancel{\begin{array}{r} 36 \\ +\,72 \\ \hline 108 \end{array}}\quad \begin{array}{r} 36 \\ +\,27 \\ \hline 63 \end{array}\quad \cancel{\begin{array}{r} 63 \\ +\,72 \\ \hline 135 \end{array}}\quad \begin{array}{r} 63 \\ +\,27 \\ \hline 90 \end{array}$$

13 problems

Secondary Sample

How many problems using addition, subtraction, multiplication, or division can be created using the numbers, 3.5, $\frac{1}{4}$, 5.75, $2\frac{2}{5}$, with an answer between 3 and 10?

$3.5 + \frac{1}{4} = 3.75$
$3.5 + 5.75 = 9.25$
$3.5 + 2\frac{2}{5} = 5.9$
$3.5 - \frac{1}{4} = 3.25$
$\cancel{3.5 - 5.75 = -2.25}$
$\cancel{3.5 - 2\frac{2}{5} = 1.1}$
$\cancel{3.5 \times \frac{1}{4} = .875}$
$\cancel{3.5 \times 5.75 = 20.125}$
$3.5 \times 2\frac{2}{5} = 8.4$
$\cancel{3.5 \div \frac{1}{4} = 14}$
$\cancel{\frac{3}{5} \div 5.75 = .6}$
$\cancel{3.5 \div 2\frac{2}{5} = 1.45}$

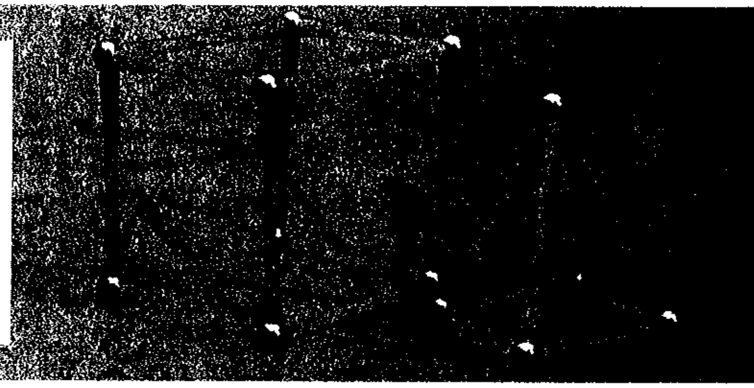
$\frac{1}{4} + 5.75 = 6$
$\cancel{\frac{1}{4} + 2\frac{2}{5} = 2\ 13/20}$
$\frac{1}{4} - 3.5 = 3.25$
$\cancel{\frac{1}{4} - 5.75 = -5.5}$
$\cancel{\frac{1}{4} - 2\frac{2}{5} = -2\frac{3}{20}}$
$\cancel{\frac{1}{4} \times 5.75 = 1.44}$
$\cancel{\frac{1}{4} \times 2\frac{2}{5} = \frac{12}{20}}$
$\cancel{\frac{1}{4} \div 3.5 = .07}$
$\cancel{\frac{1}{4} \div 5.75 = .04}$
$\cancel{\frac{1}{4} \div 2\frac{2}{5} = \frac{5}{48}}$

$5.75 + 2\frac{2}{5} = 8.15$
$\cancel{5.75 - 3.5 = 2.25}$
$5.75 - \frac{1}{4} = 5.5$
$5.75 - 2\frac{2}{5} = 3.35$
$\cancel{5.75 \times 2\frac{2}{5} = 13.8}$
$\cancel{5.75 \div 3.5 = 1.6}$
$\cancel{5.75 \div \frac{1}{4} = 23}$
$\cancel{5.75 \div 2\frac{2}{5} = 2.4}$

$\cancel{2\frac{2}{5} - 3.5 = -1.1}$
$\cancel{2\frac{2}{5} - \frac{1}{4} = 2\frac{3}{20}}$
$\cancel{2\frac{2}{5} - 5.75 = -3.35}$
$2\frac{2}{5} \div 3.5 = .7$
$\cancel{2\frac{2}{5} \div \frac{1}{4} = 9\frac{3}{5}}$
$\cancel{2\frac{2}{5} \div 5.75 = .4}$

11 problems

Working Backwards



Standard

- uses a variety of strategies in the problem-solving process

Background Information

Working backwards is a strategy to use for problems that contain linked information, where some of the information has not been provided. Usually, the missing information is at the beginning of the problem. Solving problems using this strategy requires that the problem solver start at the end of the problem and work methodically backwards until the missing information is found. This strategy is best used with students in grade 3 or higher.

There are many types of problems that can be solved using this strategy. Some of these types of problems include using the opposite operation and starting with the answer.

Procedure

Once it is decided that working backwards is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Determine what the last piece of information is in the problem. Remind students that this may be the answer in the problem.
3. Follow the steps of the problem backwards. Remind students that they may need to use the opposite (inverse) operation.
4. Check to see that the missing information in the question matches the information the students ended with.
5. Check the solution by working forward through the steps of the problem.
6. Record the solution.

Samples

The following skills and concepts illustrate how working backwards can be used with many different types of problems. Students should be comfortable with these skills in order to be able to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Working Backwards *(cont.)*

Using the Opposite Operation

Often, it is necessary to use the opposite operation when making calculations. Students need to understand how addition, subtraction, multiplication, and division are related in order to use this strategy and solve these problems correctly. Fact families are a good foundation for students before beginning this problem-solving strategy.

Grades 3–5 Sample

Taj picked some apples at the orchard during the class field trip. Sasha picked 3 more than Taj. Ernesto picked 8 less than Sasha. Jun picked 2 less than Ernesto. Ernesto picked 7 apples. How many apples did Taj pick?

Ernesto picked 7 apples.	7	
Jun picked 2 less than Ernesto.	7 – 2 = 5	The opposite operation is not needed here. Jun picked 5 apples.
Ernesto picked 8 less than Sasha.	8 + 7 = 15	The opposite operation is needed here. Sasha picked 15 apples.
Sasha picked 3 more than Taj.	15 – 3 = 12	The opposite operation is needed here. Taj picked 12 apples.

Secondary Sample

Omar has some baseball cards to trade. Amadi has 2 more than 2 times the number of baseball cards Omar has. Sean has 2 less than Amadi. Dakota has 4 less than 2 times the number of cards Sean has. Sean has 8 cards. How many cards does Omar have to trade?

Sean has 8 cards.	8	
Dakota has 4 less than 2 times the number Sean has.	$(8 \times 2) - 4 = 12$	The opposite operation is not needed here. Dakota has 12 cards.
Sean has 2 less than Amadi.	$8 + 2 = 10$	The opposite operation is needed here. Amadi has 10 cards.
Amadi has 2 more than 2 times the number Omar has.	$(10 - 2) \div 2 = 4$	The opposite operation is needed here. Omar has 4 cards.

Working Backwards (cont.)

Starting with the Answer

When the final answer is given in a problem, students can work backwards to find the information that is missing from the beginning of the problem.

Grades 3–5 Sample

Melissa is writing a paper on the mathematician John Venn. On Monday, she wrote 123 sentences about his biographical facts. On Tuesday, she wrote 96 sentences about his contributions as a mathematician. On Wednesday, she wrote 82 sentences about his anecdotes. She has to write a total of 400 sentences to complete her report. How many more sentences does she have to write?

400 – 82 – 96 – 123 = 99 more sentences

Secondary Sample

Addie started reading a library book on Monday. The book is 110 pages. On Monday, she read 20 pages. On Tuesday, she read half the number of pages she did on Monday. On Wednesday, she read 12 pages. On Thursday, she read 4 more pages than she read on Tuesday. On Friday, she read the sum of the number of pages she read on Tuesday and Wednesday. On Saturday, she read half the sum of the number of pages she read on Thursday and Friday. Today is Sunday. How many pages does she have left to read?

Monday: 20

Tuesday: 10

Wednesday: 12

Thursday: 14

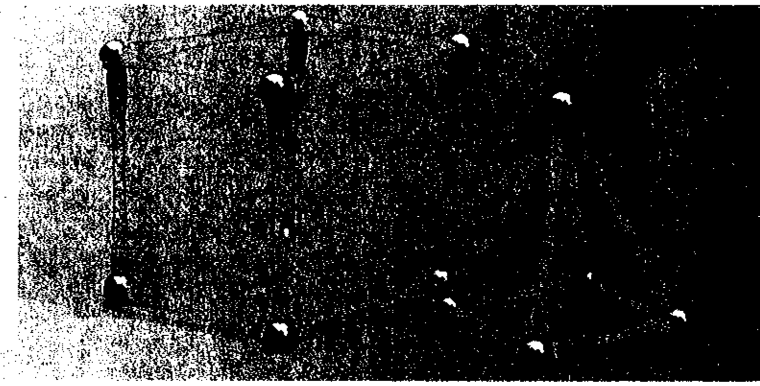
Friday: 22

Saturday: 18

Sunday: ??

110 – 20 – 10 – 12 – 14 – 22 – 18 = 14 pages to read on Sunday

Creating a Tree Diagram



Standard

- uses a variety of strategies in the problem-solving process

Background Information

Creating a tree diagram is a very visual problem-solving strategy. It is named after the way in which the information is arranged—it looks like the branches on a tree. This strategy is used to represent relationships or to show combinations of different factors in a problem. Like other problem-solving strategies, it is designed to keep the information organized in a systematic way so that parts are not left out or repeated. This strategy is best used with students in grade 3 or higher.

There are many types of problems that can be solved using this strategy. Some of these types of problems include ordering and finding combinations.

Procedure

Once it is decided that creating a tree diagram is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Decide what information is known in the problem.
3. Decide what information should be kept the same as you work through the problem.
4. Help students develop the combinations, working sequentially through the problem to create the tree diagram.
5. Reread the problem and review the diagram to note any missing or repeated information.
6. Record the solution.

Samples

The following skills and concepts illustrate how creating a tree diagram can be used with many different types of problems. Students should be comfortable with these skills in order to be able to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

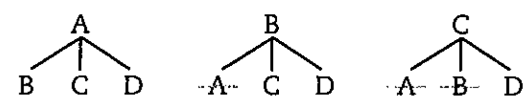
Creating a Tree Diagram *(cont.)*

Ordering

Sometimes the order of the items in a problem is key to finding the solution. Depending on the problem, there may be some possible combinations that cannot be counted as solutions. It is important to show students how to find all possible combinations first, and then cross out solutions that do not work within the parameters of the problem.

Grades 3–5 Sample

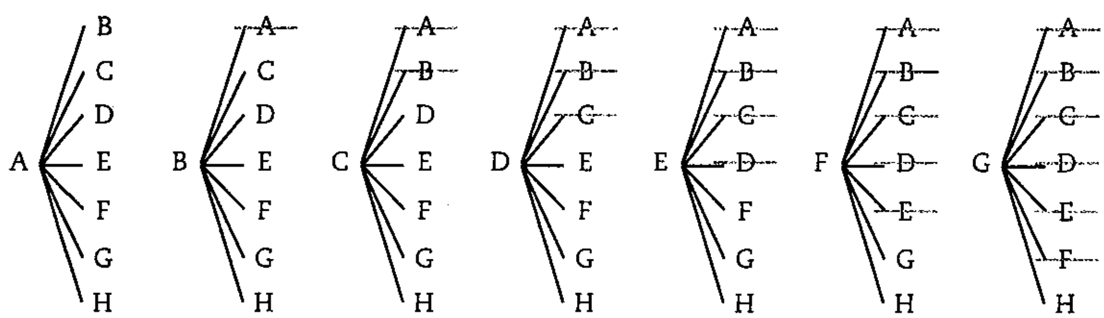
You and three of your friends are playing in a tennis tournament. Each of the players must play against each other one time. How many games will be played?



6 games played

Secondary Sample

You and seven of your friends are playing in a golf tournament. Each of the players must play against each other one time. How many games will be played?



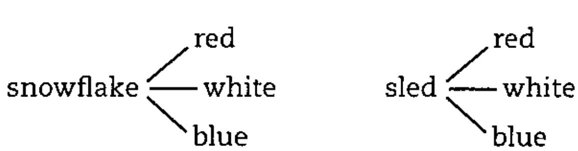
28 games played

Finding Combinations

Often the order in which the pieces of a problem are combined is not the most important factor. The solution comes when the total number of combinations is found. To do this, students must work methodically to make sure that they do not omit or duplicate combinations.

Grades 3–5 Sample

Mrs. Phillips's 4th grade class is making sugar cookies for the winter party. They will be making snowflakes and sleds with red, white, or blue icing. List all the possible combinations. If Mrs. Phillips's class wants to serve only the cookies with red and blue icing, which combination of cookies will they have for the party?



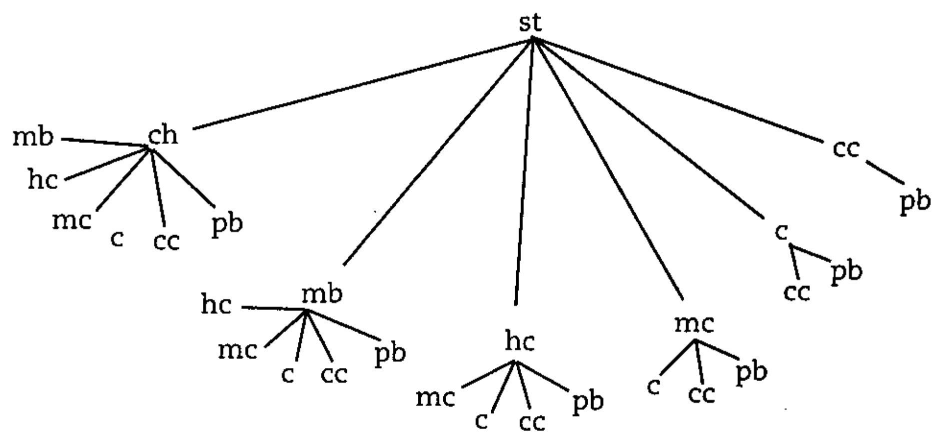
4 combinations for the party

Creating a Tree Diagram *(cont.)*

Finding Combinations *(cont.)*

Secondary Sample

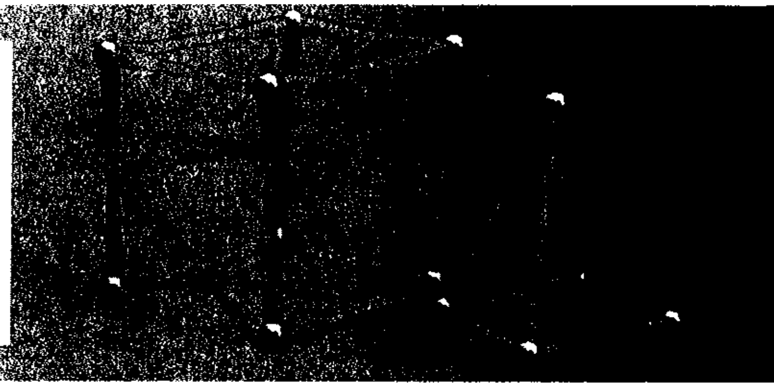
The soccer team went to an ice cream stand after its game. The flavor choices were strawberry (st), cherry (ch), mixed berry (mb), hazelnut chocolate (hc), mint chocolate (mc), coconut (c), chocolate chip (cc), and peanut butter (pb). The players each got 3 scoops of ice cream, with all 3 scoops being different flavors. List all the possible combinations if order does not matter. If Adam cannot have chocolate, how many possible combinations can he choose from?



21 different combinations

Adam can choose from 6 different combinations.

Using Simpler Numbers



Standard

- uses a variety of strategies in the problem-solving process

Background Information

Using simpler numbers is a useful strategy for solving difficult problems. In this strategy, students begin by solving the problem using an easier set of information. When the process of how to solve the problem is understood, students can solve the original problem using the scenario provided with the problem.

This problem-solving strategy can be combined with other strategies in order to understand the best solution to the more difficult problem. This strategy is best used with students in grade 3 or higher.

There are many types of problems that can be solved using this strategy. Some of these include spatial and numerical problems.

Procedure

Once it is decided that using simpler numbers is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Simplify the original problem using numbers that are easier for the students.
3. Complete the problem using the simpler numbers.
4. Help students reflect on the operations, the pattern, or the process used to solve the problem with the simpler numbers.
5. If students do not understand the general process for solving the problem, choose a different set of simpler numbers and solve the problem again. Reflect on the operations, the pattern, or the process used to solve this problem.
6. Once students can make generalizations about the process for solving the original problem, allow them to solve it using the original numbers/scenario.
7. Check the work and record the solution.

Using Simpler Numbers *(cont.)*

Samples

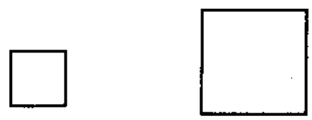
The following skills and concepts illustrate how simpler numbers can be used with many different types of problems. Students should be comfortable with them in order to be able to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Spatial

Often students are asked questions about geometric designs and patterns. To solve these types of problems, students must work out the problem using a piece of the larger set or design. Then they can replicate that answer as many times as needed to solve the original problem.

Grades 3–5 Sample

What is the area and perimeter for an 8 × 8 square?



Area = 1
Perimeter = 4

Area = 4
Perimeter = 8

Secondary Sample

Your school will use triangular tables for the end-of-year school dance. Each edge of the table can seat one person. When putting tables together, each must share one side with the other. Predict how many people can sit at 50 tables.

Number of small tables	Number [illegible]
1	3
2	4
3	5
4	6
5	7
6	8
7	9

Using Simpler Numbers *(cont.)*

Numerical

When problems involve difficult numbers, students often shy away from trying to solve them. Students can use the simpler numbers strategy to gain confidence in solving the problem without allowing the actual numbers to interfere with their reasoning process. Once students understand the process used for solving the problem, they can solve the original problem using the data given in that problem.

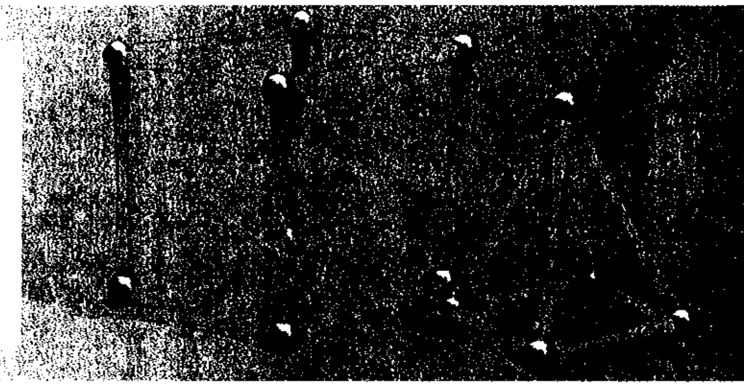
Grades 3–5 Sample

Fenway Park is home to the Boston Red Sox. As of 2005, it is the oldest American League ballpark still in operation. The park can hold 37,198 attendants. The park's sections are not numbered in numerical order. Instead, the sections are numbered 1–43, 86–97, 112, 129, 136, 148, 159, and 165. The average number of seats per section is 610. The attendants that have the digit 6 in their section number will receive an autographed baseball from the Boston Red Sox. How many sections will receive a baseball?

Secondary Sample

If it takes 6 cans of cat food to feed 5 cats for one day, how many cats can you feed for 5 days with 60 cans of cat food?

Using Logical Reasoning



Standard

- uses a variety of strategies in the problem-solving process

Background Information

On the surface, this strategy seems much like guessing and checking. However, this strategy is different because students are not making guesses to begin their problem-solving process. Instead, all of the pieces of the problem fit together like a puzzle. The logical placement of each piece of information is important in solving the problem.

Sometimes, problem solvers will have to use the information in the problem in a different order than it was presented. Inferences may also need to be made based on the information from the problem. This strategy is best used with students in grade 3 or higher.

There are many types of problems that can be solved using this strategy. Some of these types of problems include determining order and determining preferences.

Procedure

Once it is decided that using logical reasoning is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Decide on the best way to organize the information. Tell students that often a grid is needed. (It may be necessary to make grid headings or label a particular piece of the grid according to the information in the problem.)
3. Determine which information is clearly known in the problem. Record that information in the grid. Remind students that other boxes in the rows or columns on the grid may be marked based on confirmed information.
4. As information is used, have students place a line through that sentence in the problem or place a check on the first word in that sentence. This will help students keep track of the information they have recorded.
5. Examine the information that is not explicitly stated in the problem. Refer to the information on the grid when determining what inferred data should be recorded. Search each sentence for this type of data in order to complete the grid.
6. Reread the problem to make sure that the results on the grid match the information on the grid.
7. Record the final solution and answer the question from the original problem.

Using Logical Reasoning *(cont.)*

Samples

The following skills and concepts illustrate how logical reasoning can be used with many different types of problems. Students should be comfortable with these skills in order to be able to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Determining Order

Often problems will ask students to determine the order of events or objects. Placing this information in a row or grid will help students solve the problem correctly.

Grades 3–5 Sample

You are helping an engineer assemble train cars. The engineer has given you some guidelines on how to connect the cars. Using the guidelines given, connect the cars correctly.

- The types of train cars are locomotive, boxcar, hopper car, flatcar, tank car, and caboose.
- The locomotive must be first.
- The caboose must be last.
- The boxcar must be next to the caboose.
- The hopper car cannot connect to the boxcar.
- The flatcar and tank car must connect.

Secondary Sample

Kwan needs to plan his school schedule for next year. He must schedule Science from 10:00 A.M.–12:00 P.M., Study Hall at 10:00 A.M., and Gym at 1:00 P.M.

A course must be taken at the same time each day it is offered. For example, the Social Studies class can be taken from 8:00–1:00 on Tuesdays and Thursdays or from 10:00–12:00 on Tuesday and Thursdays. It cannot be taken from 8:00–10:00 on Tuesdays and from 10:00–12:00 on Thursdays.

Course	Hours Weekly	Day and Time Offered
Science	4	T/Th 10:00–12:00 or 1:00–3:00
Math	5	Daily 8:00, 9:00, 10:00, or 1:00
Study Hall	3	M/W/F 10:00 or 1:00
Language Arts	5	Daily 8:00, 10:00, 1:00, or 2:00
Lunch	5	Daily 11:00, 12:00, or 1:00
Social Studies	4	T/Th 8:00–10:00 or 10:00–12:00
Gym	3	M/W/F 8:00, 10:00, or 1:00
Computer Lab	3	M/W/F 9:00, 11:00, or 2:00
Music	2	T/Th 8:00, 10:00, 12:00, 1:00, or 2:00

Using Logical Reasoning *(cont.)*

Determining Preferences

Many logic problems discuss people's preferences for things like color, food, clothing, vehicles, and activities. These types of problems are best organized in a grid.

Grades 3–5 Sample

Antonio, Jina, Taj, and Kendra love fruit. They each have a favorite fruit: apples, oranges, strawberries, or kiwi. Use the following clues to match each with his or her favorite fruit.

- Neither of the girls likes apples.
- Only one person's favorite fruit starts with the same letter as his or her name.
- The color of Jina's favorite fruit is also her favorite color.
- The boy with the shortest name does not like apples.
- Jina wore her favorite color for picture day—red.

Secondary Sample

Rosa, Kiara, and Ashley each have a favorite sport. Use the following clues to find out which sport—basketball, tennis, or baseball—is each girl's favorite sport.

- Rosa's favorite sport is played on a court.
- Kiara's favorite sport needs equipment to hit the ball.
- Ashley's favorite sport has five players on a team.

Analyzing and Investigating

Standard

- Uses a variety of strategies in the problem-solving process

Background Information

This problem-solving strategy is an important tool for discovering solutions to real-life problems. In this strategy, students are encouraged to investigate solutions with peers and have collaborative discussions about the solution process.

Analyzing and investigating encourages students to analyze the problem, plan a strategy for solving the problem, gather needed materials and information, and complete their investigation by solving the problem. It is especially effective when students pool their knowledge and use their unique skills to help group members solve the problem. This strategy is best used with students in grade 3 or higher.

There are many skills that students will need to acquire in order to solve problems using this strategy. Some of these skills include: estimation, mental calculations, and planning and gathering information.

Procedure

Once it is decided that analyzing and investigating is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Divide the students into small groups. Have students reread the problem.
2. In groups, allow students to plan the approach they would like to use to solve the problem.
3. Students should examine the problem for important information, discussing everyone's ideas during their planning stage.
4. Allow groups to proceed with their investigations.
5. After the investigations, discuss the process used with each group separately. If the students' solution is incorrect, help them revise previous plans and construct a different approach to solving the problem. If the solution is correct, help the group reflect on its success, asking questions to break down the thought processes they used.
6. Discuss all of the group investigations as a class.

Analyzing and Investigating (cont.)

Samples

The following skills and concepts illustrate how analyzing and investigating can be used with many different types of problems. Students should be comfortable with these skills in order to be able to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Estimation

Students need to realize that initial estimates can help them plan approaches and, ultimately, find solutions to problems. Students need to realize that estimates can be used to check progress of solutions and can help them see when they have made mistakes in their calculations. To do this, students must practice simple estimation activities so that they are prepared for problem-solving scenarios.

Grades 3–5 Sample

Provide students with concrete objects and have them estimate the lengths, widths, heights, etc., of the objects. Then measure the actual objects and lead discussions about the differences between their estimations and the actual measurements.

Also ask students simple calculation problems, such as the sum of 738 and 592, that require them to round and estimate to find the answer. If students can get in the habit of estimating solutions before they do the actual calculations, this will help them gauge their answers to problems later on.

Secondary Sample

Provide students with concrete objects and have them estimate the lengths, widths, heights, etc., of the objects. Then measure the actual objects and lead discussions about the differences between their estimations and the actual measurements.

Also ask students simple calculation problems, such as the average of five different numbers, that require them to round and estimate the answer. If students can get in the habit of estimating solutions before they do the actual calculations, this will help them gauge their answers to problems later on.

Analyzing and Investigating *(cont.)*

Quick Mental Computation

Quick mental computation is another method to help students hypothesize about the outcome of a problem. Using rounded numbers, doubles, multiples, or factors to estimate a solution develops a better understanding for the actual solution and steps of the problem.

Grades 3–5 Sample

Verbally provide these students with numbers to round or double without using pencils and paper or calculators. It is also a good idea to write a sequence of numbers on the board or overhead and have students mentally compute the pattern. Make sure that the numbers and patterns are easier when first practicing this skill with students. This will give them confidence and will help them become quicker as time goes on.

Secondary Sample

Verbally provide these students with numbers to round, double, factor, or find multiples of without using pencils and paper or calculators. It is also a good idea to write a sequence of numbers on the board or overhead and have students mentally compute the pattern. Make sure that the numbers and patterns are easier when first practicing this skill with students. This will give them confidence and will help them become quicker as time goes on.

Planning an Approach to Gather Information

Planning the proper approach for gathering information and for solving a problem are difficult tasks for many students and require consistent practice. Students should decide which method will work best depending on whether the task involves observations, measurements, data, surveys, or visual diagrams. They also need to choose the best way to display their results.

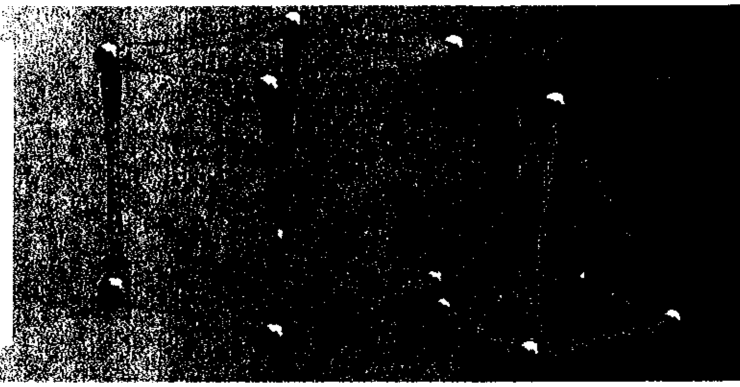
Grades 3–5 Sample

This strategy is very difficult for students at this age. You will need to model and complete "think-alouds" for students to get a better understanding of *how* to process and filter information. Provide students with opportunities to work in pairs with simpler problems at first. When students are comfortable with this, provide them with more detailed problems and require larger groups to work together.

Secondary Sample

This strategy is still difficult for many students, especially if they have not spent a significant amount of time working in groups in previous grade levels. You will need to model and show samples for students to get a better understanding of *how* to process and filter information. Provide opportunities for students to work in pairs, solving simpler problems first. When student confidence increases, challenge them with more detailed problems and increase group sizes.

Solving Open-Ended Problems



Standard

- uses a variety of strategies in the problem-solving process

Background Information

This problem-solving strategy challenges students' thinking. Use this method when problems have multiple solutions. Students need practice with this type of problem because they are accustomed to problems with a single solution. They need to realize that it is acceptable in some cases to have different answers or use diversified approaches to problem solving. This is a very authentic, real-life strategy.

It is also a unique problem-solving strategy because it allows a student to work through problems at his or her ability level using the best cognitive methods already known. Words like *create*, *investigate*, *design*, and *explore* help identify this type of problem. It is a strategy best used with students in grade 3 or higher.

There are many skills that students will need to acquire in order to solve problems using this strategy. Some of these skills include the following: using labeled or numbered counters, trying different combinations, and finding as many solutions as possible.

Procedure

Once it is decided that open-ended problem solving is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Have students decide on an approach they want to use to solve the problem.
3. Allow students freedom to use counters, extra sheets of paper, or other supplies to solve the problem.
4. Have students check their work and record their solutions.
5. Allow students to share their problem-solving process with the class and reflect together on the process and solution.

Samples

The following skills and concepts illustrate how open-ended problem solving can be used with many different types of problems. Students should be comfortable with these skills in order to be able to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Solving Open-Ended Problems (cont.)

Using Labeled or Numbered Counters

Using labeled or numbered counters can help students visualize things when problems are very involved. The counters are helpful for students when trying different solutions to the problems.

Grades 3–5 Sample

You are in charge of one of the games for the fourth-grade fair. You have 6 number counters that will have the numbers 1, 2, or 3 on them. The players will pull a number counter from a bag. The number on the counter will determine the prize. If the player pulls a number 1, he or she will win a flower eraser. If the player pulls a number 2, the prize will be a blue pencil. If the player pulls a number 3, he or she will win a pen. There are 35 flower erasers, 50 blue pencils, and 10 pens. Determine which number should go on the counters. Test the game.

Secondary Sample

You need to cover a 4 by 8 foot area with colored tiles: gold, emerald, navy, and maroon. The colored tiles are 4 inches by 4 inches. Create a pattern for the area so that the emerald tile does not touch the maroon tile.

Trying Different Combinations of Numbers

Often, students must try different combinations of numbers before finding a solution in an open-ended problem. Students need to understand that they can rearrange the combinations as many times as necessary before finding a solution.

Grades 3–5 Sample

The number of newspapers Anton sold each month is listed below.

Month	Number of Newspapers
January	28
February	31
March	34
April	29

Estimate the number of newspapers he will sell by the end of the year.

Solving Open-Ended Problems (cont.)

Trying Different Combinations of Numbers (cont.)

Secondary Sample

Pablo collects baseball cards. The number of cards he collected during each of the last four months is listed below.

Month	Number of Cards
January	18
February	21
March	17
April	19

Estimate the number of cards Pablo will have collected by the end of the year.

Finding as Many Solutions as Possible

Often, problems will ask students to find the maximum number of solutions. This can be difficult if students have never practiced this skill. Students need to understand that problems can have more than one solution and work toward finding new and creative ways to solve them.

Grades 3–5 Sample

Write a multiplication problem which has a product between 120 and 350.

Secondary Sample

Write an addition problem which includes a mixed number and two fractions and has a sum between 3 and 5. The mixed number and fractions must have different denominators.

Notes